Enrollment No: ______________________ **Exam Seat No:**______________________

C.U.SHAH UNIVERSITY

Winter Examination-2018

Subject Name: Linear Algebra-I

Subject Code: 4SC03LIA1 **Branch: B.Sc.(Mathematics, Physics))**

Semester: 3 **Date: 01/12/2018** **Time: 02:30 To 05:30** **Marks: 70**

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 **Attempt the following questions:** **(14)**

a) Define: Vector space **(02)**

b) State Cauchy-Schwarz's inequality. **(02)**

c) Define: Basis **(01)**

d) Intersection of two subspaces is subspace.-True or False? **(01)**

e) $\dim(P_2) =$ _____ . **(01)**

f) For a bijective mapping $T : R^3 \to R^3$ then the rank of T is _____. **(01)**

g) $T = \begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix}$ is the matrix form for shear in the y-direction on R^2.- True or False? **(01)**

h) Define: Norm in inner product space **(01)**

i) Find the value of k for which the vectors $(1,0,0),(0,2,0)$ and $(0,0,k)$ are linearly dependent. **(01)**

j) Find $d(u,v)$ if $u=(u_1,u_2)=(5,4); v=(v_1,v_2)=(2,6)$ and inner product space is define by $\langle u,v\rangle = 3u_1v_1 + 2u_2v_2$ **(01)**

k) Suppose $T : V \to W$ is a linear transformation where $\dim(V) = \dim(W)$. Then which of the following is true? **(01)**

a) T is injective b) T is invertible c) T is surjective d) none of these

l) Let $T : V \to W$ be a linear transformation. What is the rank-nullity formula? **(01)**

a) $\dim(V) = rank(T) - nullity(T)$
b) $\dim(W) = rank(T) + nullity(T)$
c) $\dim(W) = rank(T) - nullity(T)$
d) $\dim(V) = rank(T) + nullity(T)$

Attempt any four questions from Q-2 to Q-8

Q-2 **Attempt all questions** **(14)**

a) Let $V = \{f(x): f(x) = a_0 + a_1x + a_2x^2 + \cdots + a_nx^n, a_i \in \boldsymbol{R}\}$ and $F = \boldsymbol{R}$ with the usual addition and scalar multiplication on V show that V is a vector space **(07)**

over $\boldsymbol{R}$.

b) Define: Subspace of Vector space. **(07)**

Also Check whether the following are subspaces of vector space V.

i) $W=\{(x,y,z) \mid x^2+y^2+z^2 \le 1\}; \quad V=R^3$

ii) $W=\{(x,y,z) \mid x-3y+4z=0\}; \quad V=R^3$

iii) $W=\left\{\begin{bmatrix} a & b \\ c & d \end{bmatrix} \middle| a+b+c+d=0\right\}; \quad V=M_{22}$

Q-3 **Attempt all questions** **(14)**

a) Which of the following are linear transformations? **(06)**

i) $T:R^2 \to R; \quad T(x,y)=xy$

ii) $T:R^2 \to R^2; \quad T(x,y)=(x+2y,3x-y)$

b) Prove that the set $S=\{(1,2,1),(2,1,1),(1,1,2)\}$ is Basis of $\boldsymbol{R}^3$. **(04)**

c) Express $(3,4,6)$ as a linear combination of $\{v_1,v_2,v_3\}$, Where **(04)**

$v_1=(1,-2,2), v_2=(0,3,4), v_3=(1,2,-1)$.

Q-4 **Attempt all questions** **(14)**

a) Consider the basis $S=\{v_1,v_2\}$ for R^2, where $v_1=(1,1), v_2=(1,0)$ and and let $T:R^2 \to R^2$ be the linear transformation such that $T(v_1)=(1,-2)$ and $T(v_2)=(-4,1)$. Find $T(x,y)$ and $T(5,-3)$. **(05)**

b) If $T:R^2 \to R^2$ defined by $T(x,y)=(2x-y,8x+4y)$ then find the range of T, rank of T, Ker (T) and nullity of T. **(05)**

c) Find the domain and co-domain of $T_2{}^\circ T_1$ and find $(T_2{}^\circ T_1)(x,y)$. **(04)**

a. $T_1(x,y)=(x-3y,0)$ and $T_2(x,y)=(4x-5y,3x-6y)$

b. $T_1(x,y)=(2x,-3y,x+y)$ and $T_2(x,y)=(x-y,y+z)$

Q-5 **Attempt all questions** **(14)**

a) Let $T:R^3 \to R^3$ be the L.T. defined by $T(x,y,z)=(x+2y+z,2x-y,2y+z)$. **(07)**

Find the matrix representation of T with respect to basis

(i) S_1

(ii) S_1 and S_2

(iii) S_2 and S_1,

where $S_1=\{(1,0,0),(0,1,0),(0,0,1)\}$ and $S_2=\{(1,0,1),(0,1,1),(0,0,1)\}$ for R^3.

b) Prove that linear transformation $T:R^3 \to R^3$ is isomorphism. **(04)**

Where $T(x,y,z)=(x+3y,y,2x+z)$

c) Explain: Reflection operators **(03)**

Q-6 **Attempt all questions** **(14)**

a) Which of the following set S of vectors/polynomials in vector space V are **(06)**

linearly dependent or linearly independent?

i) $S=\{(4,-1,2),(-4,10,2),(4,0,1)\}$; $V=R^3$

ii) $S=\{2+x+x^2, x+2x^2, 2+2x+3x^2\}$; $V=P_2$

b) Let $T:R^3 \to R^3$ be linear transformation defined by $T(x_1,x_2,x_3)=(x_1-x_2+x_3, 2x_2-x_3, 2x_1+3x_2)$ determine whether T is one-one. If so find $T^{-1}(x_1,x_2,x_3)$. **(06)**

c) Check whether the set $V=\{(x,e^x): x>0\}$ is a vector space or not with the given operation: $(x,e^x)+(y,e^y)=(x+y,e^{x+y})$ and $(x,e^x)=(\alpha x, e^{\alpha x})$. **(02)**

Q-7 **Attempt all questions** **(14)**

a) Let S be a finite set of vectors in a vector space V over a field F. The set of all linear combinations of the vectors in S forms a smallest subspace of V containing S. **(06)**

b) If V and W are vector spaces over a field F and $T:V\to W$ a linear transformation then $Ker(T)$ is a subspace of V. **(04)**

c) Find the space generated by $v_1=(1,3,0)$, $v_2=(2,1,-2)$. Examine if $v_3=(-1,2,3)$, $v_4=(4,7,-2)$ are in the space. **(04)**

Q-8 **Attempt all questions** **(14)**

a) Find the cosine angle between given vectors and also verify Cauchy-Schwarz inequality for $u=(1,0,1,0)$ & $v=(-3,-3,-3,-3)$. **(05)**

b) Find $\langle f,g\rangle$, $\|f\|$ and $\|g\|$, if $f(x)=3x-5$ and $g(x)=x^2+1$ and the inner product is defined by $\langle f,g\rangle=\int_0^1 f(x)g(x)\,dx$. **(05)**

c) Prove that $\langle u,v\rangle=2u_1v_1+u_2v_2+4u_3v_3$ is an inner product space on R^3. **(04)**